Non-ideal effects on the typical trailing edge shock pattern of ORC turbine blades

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MOTIVATIONS

ORC attractive features
- Adaptability to various (low temperature) heat sources
- Lower complexity wrt steam cycle
- Turbine technical advantages wrt steam turbine (lower rmp, lower pressures, no erosion)
- High flexibility
- …

ORC challenges
- Choice of suitable working fluid
- Transient phenomena
- Complex thermodynamic modelling of the working fluid
- Heat exchangers and turbine design
- …
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MOTIVATIONS

ORC turbine

- Typically few stages (often one only)
- High pressure ratio

- Design expansion through the non-ideal regime: low values of the speed of sound → highly supersonic flow
- Shock waves: fish-tail shocks, post-expansion, off-design
- Large contribution of inviscid loss to total loss

Research question

How do non-ideal effects across oblique shocks impact on the design of ORC turbines?
➔ Introduction: NICFD

➔ Methodology

➔ Oblique shocks in the non-ideal flow regime

➔ Application: oblique shocks in siloxane MDM

➔ Discussion and concluding remarks
Non-Ideal Compressible Fluid Dynamics

\[ P_v \neq RT \]

Features
- Subject: dense vapours, supercritical fluids, two-phase compressible flows
- Compressibility
- Phase transition
- Critical point

Application
- ORC
- Supercritical CO\(_2\)
- Refrigeration
- Oil & Gas compression/expansion
- ...

Fluid: MDM (RefProp)
INTRODUCTION

Non-Ideal Compressible Fluid Dynamics

\[ P v \neq RT \]

Measure of non-ideality in compressible flows:

The fundamental derivative of gasdynamics (Thompson 1971)

\[ \Gamma = 1 + \frac{\rho}{c} \left( \frac{\partial c}{\partial \rho} \right)_s = 1 + \frac{\rho}{c} \left( \frac{\partial^2 P}{\partial \rho^2} \right)_s = \frac{v^3}{2c^2} \left( \frac{\partial^2 P}{\partial v^2} \right)_s \]

Gasdynamic regimes

- \( \Gamma > 1 \) Ideal regime
- \( \Gamma < 1 \) Non-Ideal regime
  - \( 0 < \Gamma < 1 \) Non-Ideal classical regime
  - \( \Gamma < 0 \) Non-Classical regime

Fluid: MDM (RefProp)
Rankine-Hugoniot relations

\[ h_A - \frac{1}{2} P_A (v_A + v_B) = h_B - \frac{1}{2} P_B (v_A + v_B) \]

\[ \sqrt{-\frac{(P_B - P_A)}{(v_B - v_A)}} = \rho_A |u_A| \sin \beta \]

\[ \rho_A \tan \beta = \rho_B \tan (\beta - \theta) \]

\[ |u_A| \cos \beta = |u_B| \cos (\beta - \theta) \]

Admissibility conditions

\[ s_B > s_A \]

\[ \left( M_{nB} = \frac{|u_{nB}|}{c_B} < 1 < \frac{|u_{nA}|}{c_A} = M_{nA} \right) \]
Deflection shock polars: X-θ diagrams for fixed upstream state

Shock angle polar

Mach number polar

A – Mach wave (acoustic limit)
N – normal shock

D – detachment point (max deflection)
S – downstream sonic point
Perfect-gas: explicit formulas

\[ \tan \theta = \frac{2}{\tan \beta} \left[ \frac{M_A^2 \sin^2 \beta - 1}{M_A^2 (\gamma + \cos 2\beta) + 2} \right] \]

\[ \frac{\rho_B}{\rho_A} = \frac{(\gamma + 1)M_A^2 \sin^2 \beta}{2 + (\gamma - 1)M_A^2 \sin^2 \beta} \]

\[ \frac{P_B}{P_A} = 1 + \frac{2\gamma}{\gamma + 1} (M_A^2 \sin^2 \beta - 1) \]

\[ M_B^2 = \frac{1}{\sin^2 (\beta - \vartheta)} \frac{1 + \frac{\gamma - 1}{2} M_A^2 \sin^2 \beta}{\gamma M_A^2 \sin^2 \beta - \frac{\gamma - 1}{2}} \]

Dependencies:
- Deflection angle \( \vartheta \)
- Upstream Mach number \( M_A \)

No dependence on the upstream thermodynamic state (e.g. \( P_A, \rho_A \))
Non-ideal regime: acoustic limit

\[ \beta = \sin^{-1}(1/M_A) + \frac{\Gamma_A}{2} \frac{M_A^2}{M_A^2 - 1} \vartheta + O(\vartheta^2) \]

\[ \frac{\rho_B}{\rho_A} = 1 + \frac{M_A \Gamma_A}{\sqrt{M_A^2 - 1}} \vartheta + O(\vartheta^2) \]

\[ \frac{P_B}{P_A} = 1 + \frac{\rho_A c_A^2}{P_A} \frac{M_A \Gamma_A}{\sqrt{M_A^2 - 1}} \vartheta + O(\vartheta^2) \]

\[ M_B = M_A + \left(1 - \Gamma_A - \frac{1}{M_A^2}\right) \frac{M_A^3}{\sqrt{M_A^2 - 1}} \vartheta + O(\vartheta^2) \]

Dependences:
- Deflection angle \( \vartheta \)
- Upstream Mach number \( M_A \)
- Upstream thermodynamic state (e.g., \( P_A, \rho_A \))
APPLICATION: OBLIQUE SHOCKS IN MDM

Parametric study

- Fluid: siloxane MDM (RefProp)
- Fixed upstream entropy in the non-ideal thermodynamic regime ($\Gamma < 1$)
- Fixed upstream Mach number ($M_A = 2$)
\[ \beta = \sin^{-1}(1/M_A) + \frac{\Gamma_A}{2} \frac{M_A^2}{M_A^2 - 1} \theta \]

dependence on the upstream tmd state through \( \Gamma_A \)

- Acoustic limit:
- Strong dependence of the detachment angles on the upstream tmd state


\[ \beta = \sin^{-1}\left(\frac{1}{M_A}\right) + \frac{\Gamma_A}{2} \frac{M^2_A}{M^2_A - 1} \vartheta \]

- Acoustic limit:
- Strong dependence of the detachment angles on the upstream tmd state.
APPLICATION: OBLIQUE SHOCKS IN MDM

$M_B - \theta$ Diagram

- Acoustic limit:

$$M_B = M_A + \left(1 - \Gamma_A - \frac{1}{M_A^2}\right)\frac{M_A^3}{\sqrt{M_A^2 - 1}}\theta$$

dependence on the upstream tmd state through $\Gamma_A$

- Non-ideal oblique shocks (Mach number increasing):

$$\Gamma_A < 1 - \frac{1}{M_A^2}$$

for small deviations
APPLICATION: OBLIQUE SHOCKS IN MDM

$P_B^t/P_A^t - \vartheta$ Diagram

- Fixed $\vartheta \rightarrow$ non-monotonic variation of the shock loss with the upstream pressure
- Larger shock loss across strong oblique shocks w.r.t perfect-gas case
- Smaller shock loss across weak oblique shocks w.r.t perfect-gas case for low values of $\Gamma_A$
Further parametric study

- Same fluid, same isentrope, $M_A = 1.5$

- Non-ideal oblique shocks (Mach number increasing): lower threshold on the upstream Mach number

$$M_{A,\text{min}} = 1/\sqrt{1 - \Gamma_{A,\text{min}}}$$
Extension to other fluids

- Same qualitative behaviour expected for most moderate-to-high molecularly complex fluids

- Qualitatively similar thermodynamic topology of the fundamental derivative of gasdynamics

\[
\Gamma
\]

\[
v/v_c
\]
Extension to other fluids

- Non-ideal oblique shocks (Mach number increasing): lower threshold on the upstream Mach number
  \[ M_{A,\text{min}} = \frac{1}{\sqrt{1 - T_{A,\text{min}}}} \]

- Total conditions for non-ideal oblique shocks may exceed thermal stability limit

Example: minimum \( P^t \) and \( T^t \) for non-ideal oblique shocks along isentrope tangent to VLE

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( P_{\text{min}}^t ) [bar]</th>
<th>( T_{\text{min}}^t ) [°C]</th>
<th>( T_{\lim}^t ) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDM</td>
<td>16.55</td>
<td>299.0</td>
<td>( \sim 290 \div 300 )</td>
</tr>
<tr>
<td>MM</td>
<td>25.90</td>
<td>262.0</td>
<td>( \sim 300 )</td>
</tr>
<tr>
<td>Toluene</td>
<td>74.00</td>
<td>355.0</td>
<td>( \sim 400 )</td>
</tr>
<tr>
<td>Isopentane</td>
<td>64.80</td>
<td>221.0</td>
<td>( \sim 290 )</td>
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<tr>
<td>Cyclopentane</td>
<td>97.30</td>
<td>280.7</td>
<td>( \sim 275 )</td>
</tr>
<tr>
<td>R245fa</td>
<td>107.77</td>
<td>204.0</td>
<td>( \sim 300 )</td>
</tr>
</tbody>
</table>
CONCLUSIONS

- Oblique shock waves were investigated in the non-ideal regime because of their relevance to ORC turbine flows.

- Main results:
  - Shock angle polar shifts to higher deflection angle
  - Appearance of Mach number-increasing oblique shocks (non-ideal oblique shocks)
  - Shock loss: larger across strong oblique shocks, possibly smaller across weak oblique shocks w.r.t. perfect-gas case

- MDM used for explanatory purposes, direct extension to other molecularly complex fluids employed in ORCs

- Highly non-ideal effects at design conditions only for supercritical ORCs
FUTURE WORK

- Numerical investigation on real vanes configurations at design and off-design conditions

- Experimental observation of non-ideal effects across oblique shock waves at TROVA (Test Rig for Organic Vapours), CREALab PoliMi
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Thanks for your attention!

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